

TURTLE, G4beamline, Rectangular Bends, and Fringe Fields

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Overview

- This is a comparison of the methods used by TURTLE and G4beamline.
- It is not an attempt to defend the physics model of either.

TURTLE

- “Trace Unlimited Rays Through Lumped Elements”
- Uses matrix optics, up to third-order, to trace individual rays through an optics system.
- Refer to D.C. Carey, et al., “TURTLE with MAD Input”, SLAC-R-544/Fermilab-Pub-99/232

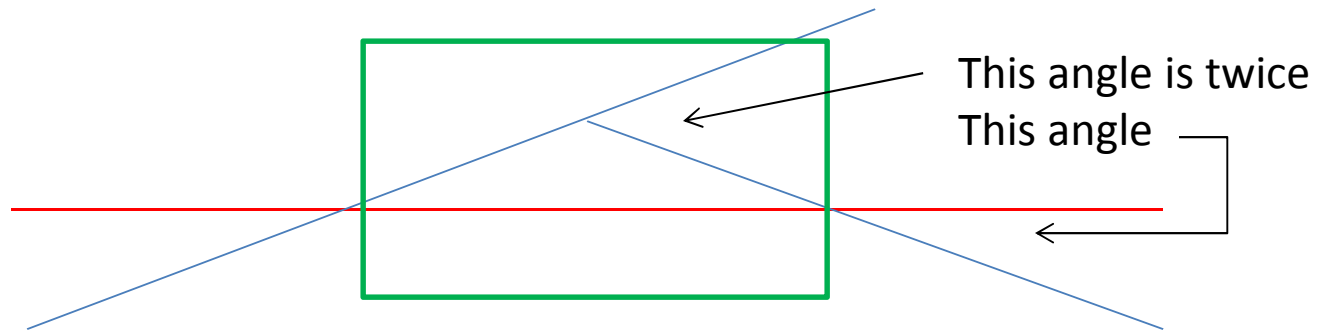
G4Beamline

- “G4Beamline is a particle tracking and simulation program based on the Geant4 toolkit ...”
- Tom Roberts, “G4beamline User’s Guide”

Rectangular Bend

“What It Is”

- Bending magnet is a rectangle – although it can be curved, as long as ...
- Angle between central ray and a line normal to face of magnet is one-half bend angle.



Rectangular Bend

Difference in Models

TURTLE:

- At this point, fringe fields are ignored.
- TURTLE treats a rectangular bend as a sector bend with rotated pole faces. The pole-face rotation leads to a focusing term – defocusing in the bend plane, focusing in the non-bend plane.
- The pole-face rotation is treated as a hard-edge, i.e., the field ends abruptly at the end of the magnet.
- A new coordinate is introduced which extends perpendicular to the (sector) pole face and is defined through the rotated face.
- This gives rise to the defocusing term in the bend plane – which exactly cancels the focusing term a sector bend would have.
- The “extra” field, along with Maxwell’s equations, give rise to a term in the non-bend plane. Integrating through this results in a focusing term equal to the (bend plane) focusing term.
- For details, refer to David C. Carey, “The Optics of Charged Particle Beams”.

Rectangular Bends

Difference in Models

G4beamline

- At this point, fringe fields are ignored.
- Treats a rectangular bend as a magnetic field perpendicular to bend plane.
- Due to geometry of a uniform field in a rectangular region, there is no bend-plane focusing.
- Because there is no bend-plane magnetic field component, there no non-bend-plane focusing.
- $\vec{v} \times \vec{B} = v_z * B_y = F_x$

Rectangular Bends

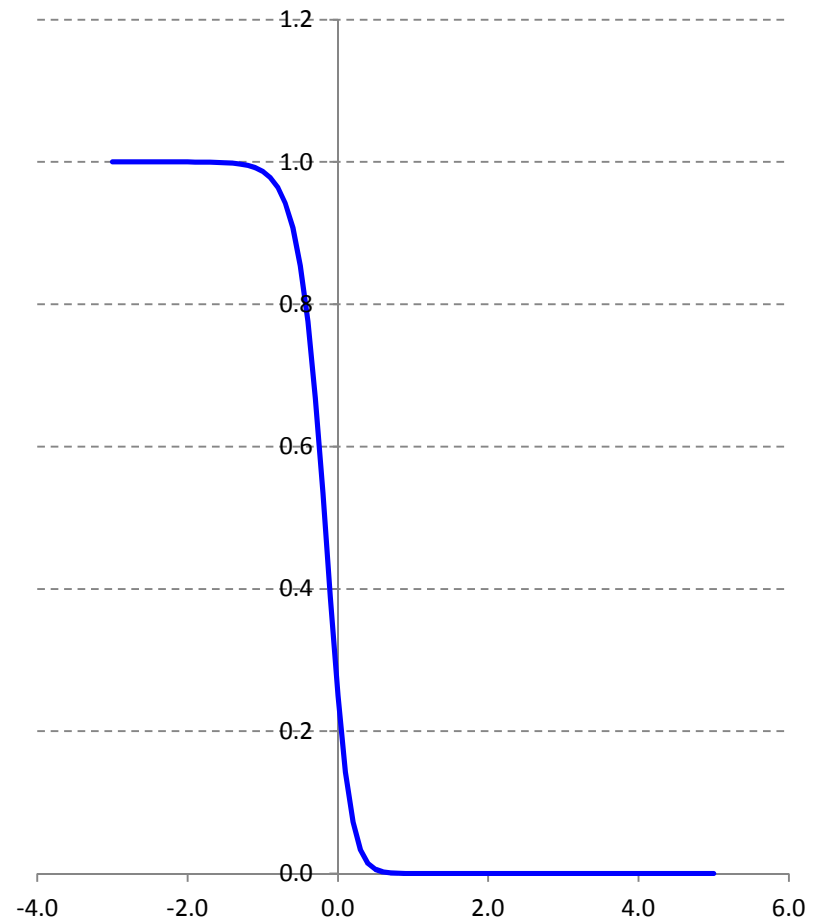
Summary

- A “rectangular bend without fringe fields” is modeled differently in TURTLE and G4beamline.
- This is more than a statement about the mathematics (linear algebra versus numeric integration).
- The *physics model* differs.
- Or, as Doug observed, $0 \neq 0$!

Fringe Field

“What It Is”

- The “fringe field” is the magnetic field that “sticks out of the magnet”.
- However, it also “sticks into the magnet”, that is, component perpendicular to the bend plane begins to decrease before the edge of the pole tip.



Fringe Field TURTLE

- Assume an effective boundary defined by:

$$\int_{-\sigma_1}^{\infty} B_y(\sigma) = \sigma_1 B_0$$

That is, the integrated field beyond the effective boundary equals the missing integrated field inside the boundary, up to some point.

- Lots of math, find that the pole face is effectively rotated by an additional amount

$$\varphi = \frac{g}{\rho} \frac{1 + \sin^2 \beta}{\cos \beta} I_2$$

Where g = gap, ρ =radius of curvature, β =one-half bend angle.

- What is I_2 ?

Fringe Field TURTLE

- $I_2 = \int_{-\sigma_1}^{\infty} \frac{B(\sigma)_y (B_0 - B(\sigma)_y)}{g B_0^2} d\sigma$
- Well, that's real clear ...

Fringe Fields

G4beamline

- Assumes magnetic field satisfies Maxwell's equations to first order, and the field perpendicular to the bend plane can be characterized by

$$B_y(z) = F(z)B_0$$

And

$$F(z) = \frac{1}{1 + e^{\sum_{i=0}^n a_i \left(\frac{z}{g}\right)^i}}$$

- This form should be used for $-3 \leq \frac{z}{g} \leq 5$
- Let $\sigma = \frac{z}{g}$, $F(z) \rightarrow F(\sigma)$

Fringe Fields

Tying it Together

- Now, we can re-write the equation for I_2 (TURTLE) in terms of F (G4beamline).

$$\begin{aligned} I_2 &= \int_{\sigma_1}^{\sigma_2} \frac{F(\sigma) B_0 (B_0 - F(\sigma) B_0)}{g B_0^2} d\sigma \\ &= \frac{1}{g} \left[\int_{\sigma_1}^{\sigma_2} F(\sigma) d\sigma - \int_{\sigma_1}^{\sigma_2} F(\sigma) d\sigma \right] \end{aligned}$$

- We have a way to compare TURTLE to G4beamline if we know F .

Problems

- What is F ?
- Our magnets typically have $g = 180mm$ and $L = 800mm$, so $L/g \approx 4.5$, so integrating three gap-widths from either side may be problematic (depends on how “flat” field is in central region).
- Need to measure or simulate field.